



# Production and Cost Estimation

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## Learning Objectives

After reading Chapter 10 and working the problems for Chapter 10 in the textbook and in this Student Workbook, you should be able to:

- Specify and estimate a short-run production function using a cubic specification of the production function.
- Specify and estimate a short-run cost function using a cubic specification.

In order to accomplish these goals, Chapter 10 shows you how to:

- Estimate the parameters of a cubic short-run production function by using the technique of regression through the origin.
- Find the region of diminishing returns.
- Estimate the output level at which  $AVC$  reaches its minimum value.
- Estimate the parameters of a cubic short-run total variable cost equation along with the associated average variable cost and marginal cost equations.

## Essential Concepts

1. The cubic empirical specification for a *short-run* production function is derived from a *long-run* cubic production function. The cubic form of the *long-run* production function is expressed as

$$Q = aK^3L^3 + bK^2L^2$$

2. The properties of a short-run cubic production function ( $Q = AL^3 + BL^2$ ) are:

- a. Holding capital constant at  $\bar{K}$  units, the short-run cubic production function is derived as follows:

$$\begin{aligned} Q &= a\bar{K}^3L^3 + b\bar{K}^2L^2 \\ &= AL^3 + BL^2 \end{aligned}$$

where  $A = a\bar{K}^3$  and  $B = b\bar{K}^2$

- b. The average and marginal products of labor are respectively

$$AP = Q/L = AL^2 + BL \quad \text{and} \quad MP = \Delta Q/\Delta L = 3AL^2 + 2BL$$

- c. Marginal product of labor begins to diminish beyond  $L_m$  units of labor and average product of labor begins to diminish beyond  $L_a$  units of labor, where

$$L_m = -\frac{B}{3A} \quad \text{and} \quad L_a = -\frac{B}{2A}$$

- d. In order to have the necessary properties of a production function, the parameters must satisfy the following restrictions:

$$A < 0 \quad \text{and} \quad B > 0$$

3. To estimate a cubic short-run production function using linear regression analysis, you must first transform the cubic equation into linear form:

$$Q = AX + BW$$

where  $X = L^3$  and  $W = L^2$ . In order to correctly estimate the cubic equation, the estimated regression line must pass through the origin; that is, when  $L = 0$ ,  $Q = 0$ . *Regression through the origin* simply requires the analyst to specify in the computer routine that the origin be suppressed.

4. Short-run cost functions should be estimated using data for which the level of usage of one or more of the inputs is fixed. Usually time-series data for a specific firm are used to estimate short-run cost functions.
5. Collecting data may be complicated by the fact that accounting data are based on expenditures and may not include the firm's opportunity cost of using the various inputs. In particular, capital costs should reflect not only acquisition cost but also the rental income forgone by using (rather than renting) the capital, the depreciation, and any capital gain or loss.
6. The effects of inflation on cost data must be eliminated. To adjust nominal cost figures for inflation, divide each observation by the appropriate price index for that time period.
7. The properties of a short-run cubic cost function ( $TVC = aQ + bQ^2 + cQ^3$ ) are:

- a. The average variable cost and marginal cost functions are, respectively,

$$AVC = a + bQ + cQ^2 \quad \text{and} \quad SMC = a + 2bQ + 3cQ^2$$

- b. Average variable cost reaches its minimum value at  $Q_m = -b/2c$ .
- c. To conform to the theoretical properties of a cost function, the parameters must satisfy the following restrictions:

$$a > 0, b < 0, \text{ and } c > 0$$

- d. The cubic specification produces an *S*-shaped *TVC* curve and *U*-shaped *AVC* and *SMC* curves.
- e. Because all three cost curves (*TVC*, *AVC*, and *SMC*) employ the same parameters, it is only necessary to estimate any one of these functions in order to obtain estimates of all three curves.
- f. In the short-run cubic specification, input prices are assumed to be constant and are not explicitly included in the cost equation.

## Summary of Short-Run Empirical Production and Cost Functions

<i>Short-run cubic production equations</i>	
Total product	$Q = AL^3 + BL^2$
Average product of labor	$AP = AL^2 + BL$
Marginal product of labor	$MP = 3AL^2 + 2BL$
Diminishing marginal returns	beginning at $L_m = -\frac{B}{3A}$
Restrictions on parameters	$A < 0$ and $B > 0$

  

<i>Short-run cubic cost equations</i>	
Total variable cost	$TVC = aQ + bQ^2 + cQ^3$
Average variable cost	$AVC = a + bQ + cQ^2$
Marginal cost	$SMC = a + 2bQ + 3cQ^2$
Average variable cost reaches minimum at	$Q_m = -\frac{b}{2c}$
Restrictions on parameters	$a > 0, b < 0, c > 0$

## Matching Definitions

empirical production function  
 long-run production function  
 short-run production function  
 cubic production function  
 short-run cubic production function

regression through the origin  
 nominal cost data  
 deflating  
 user cost of capital

1. \_\_\_\_\_ The exact mathematical form of the equation to be estimated.
2. \_\_\_\_\_ Production function in which all inputs are considered variable.
3. \_\_\_\_\_ Production function in which at least one input is fixed.
4. \_\_\_\_\_ Production function of the form  $Q = aK^3L^3 + bK^2L^2$ .
5. \_\_\_\_\_ Production function of the form  $Q = AL^3 + BL^2$ .
6. \_\_\_\_\_ A regression with the intercept parameter suppressed.
7. \_\_\_\_\_ Data that have not been corrected for the effects of inflation.
8. \_\_\_\_\_ The process of correcting for inflation by dividing nominal data by a price index.
9. \_\_\_\_\_ The firm's opportunity cost of using capital.

## Study Problems

1. Name the following empirical specifications of production and cost functions:
  - a.  $TVC = aQ + bQ^2 + cQ^3$  \_\_\_\_\_
  - b.  $SMC = a + 2bQ + 3cQ^2$  \_\_\_\_\_
  - c.  $Q = aK^3L^3 + bK^2L^2$  \_\_\_\_\_
  - d.  $AVC = a + bQ + cQ^2$  \_\_\_\_\_
  - e.  $Q = AL^3 + BL^2$  \_\_\_\_\_
2. What restrictions must be placed on the parameters in the empirical production and cost functions in question 1 above?
3. A firm estimates its long-run production function to be

$$Q = -0.008K^3L^3 + 10K^2L^2$$

Suppose the firm employs 15 units of capital.

- a. The equations for the product curves in the short run are:

$$TP = \underline{\hspace{4cm}}$$

$$AP = \underline{\hspace{4cm}}$$

$$MP = \underline{\hspace{4cm}}$$

- b. At \_\_\_\_\_ units of labor, marginal product of labor begins to diminish.  
 c. At \_\_\_\_\_ units of labor, average product of labor begins to diminish.  
 d. Calculate the marginal product and average product of labor when 20 units of labor are employed.

$$MP_{L=20} = \underline{\hspace{2cm}}$$

$$AP_{L=20} = \underline{\hspace{2cm}}$$

4. A firm estimates its cubic production function of the following form

$$Q = AL^3 + BL^2$$

and obtains the following estimation results:

DEPENDENT VARIABLE:	Q	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	62	0.7032	142.175	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT					
L3		-0.050	0.013	-3.85	0.0003
L2		0.600	0.250	2.40	0.0195

The firm pays \$36 per unit for labor services.

- a. The estimated total, average, and marginal product functions are:

$$Q = \underline{\hspace{2cm}}$$

$$AP = \underline{\hspace{2cm}}$$

$$MP = \underline{\hspace{2cm}}$$

- b. Are the parameters of the correct sign and are they significant? Discuss the  $p$ -values.  
 c. Average product reaches its maximum value at \_\_\_\_\_ units of labor.  
 d. Average product reaches its maximum value at \_\_\_\_\_ units of output.  
 e. At the output level for part  $d$ ,  $AVC = \$$  \_\_\_\_\_ and  $SMC = \$$  \_\_\_\_\_.  
 f. When labor usage is 7 units,  $AVC = \$$  \_\_\_\_\_ and  $SMC = \$$  \_\_\_\_\_.

5. Consider a firm that estimates the following average variable cost function:

$$AVC = a + bQ + cQ^2$$

The computer printout for the regression analysis is:

DEPENDENT VARIABLE:	AVC	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	16	0.9000	58.50	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		75.00	25.00	3.00	0.0102
Q		-2.40	0.40	-6.00	0.0001
Q2		0.06	0.20	3.00	0.0102

- Determine whether the estimate values of the coefficients indicate a U-shaped  $AVC$  curve at the 5 percent level of significance.
- The marginal cost function associated with this  $AVC$  function is  $SMC =$  \_\_\_\_\_.
- The total variable cost function associated with this function is  $TVC =$  \_\_\_\_\_.
- $AVC$  reaches its minimum value at  $Q_m =$  \_\_\_\_\_.
- Minimum  $AVC =$  \$ \_\_\_\_\_.

## Computer Problem

Mercantile Metalworks, Inc. manufactures wire carts for grocery stores. The production manager at Mercantile wishes to estimate an empirical production function for the assembly of carts using the following time-series data for the last 22 days of assembly operations.  $L$  is the daily number of assembly workers employed, and  $Q$  is the number of carts assembled (completely) for that day. Mercantile pays its assembly workers \$160 per day in wages and benefits.

Day	Number of workers $L$	Number of carts assembled $Q$	Day	Number of workers $L$	Number of carts assembled $Q$
1	15	75	12	40	2,165
2	21	897	13	21	1,534
3	24	1,280	14	27	835
4	32	1,251	15	20	906
5	36	1,315	16	15	102
6	38	2,837	17	36	1,424
7	18	590	18	14	111
8	18	129	19	24	868
9	41	1,572	20	25	916
10	36	2,005	21	32	1,341
11	44	1,024	22	21	806

- Use a computer regression package or Excel to estimate the following short-run cubic production function:

$$Q = AL^3 + BL^2$$

Do the parameter estimates have the appropriate algebraic signs? Are they statistically significant at the 1 percent level of statistical significance? How well did the empirical model do in explaining the variation in the number of carts assembled each day?

2. What are the estimated total, average, and marginal product functions from your regression results in Part 1?
3. At what level of labor usage does average product reach its maximum value? In a day, how many carts per worker are assembled when average product is maximized? What is average variable cost when average product is maximized?
4. What is short-run marginal cost when average product is maximized?
5. Beyond what level of *labor employment* does the law of diminishing returns set in? Beyond what level of *output*?

## Multiple Choice / True-False

1. Empirical production and cost functions
  - a. can be obtained using regression analysis.
  - b. require data from actual production operations.
  - c. can be used in making profit-maximizing decisions.
  - d. are curvilinear functions that can be estimated using regression analysis.
  - e. all of the above.
2. Time-series data for a specific firm are often used to estimate short-run cost functions because
  - a. over the chosen period of time, a firm will not be able to vary the usage of one or more inputs.
  - b. cross-section data would probably include firms with different levels of capital usage.
  - c. time-series data are best suited for investment decisions.
  - d. both *a* and *b*.
  - e. both *b* and *c*.
3. A cubic specification for a short-run cost function is appropriate when the scatter diagram indicates
  - a. an *S*-shaped short-run marginal cost curve.
  - b. total cost increases at an increasing rate throughout the range of output.
  - c. an *S*-shaped short-run total variable cost curve.
  - d. an *S*-shaped short-run average total cost curve.
  - e. a *U*-shaped short-run total cost curve.
4. The user cost of capital includes
  - a. acquisition cost.
  - b. depreciation from the use of capital.
  - c. capital gains or losses.
  - d. revenue foregone by using rather than renting the capital.
  - e. all of the above.

5. To adjust cost data for the effects of inflation,
  - a. throw out the observations that occur in years with high inflation rates.
  - b. deflate cost figures by dividing by an appropriate price index.
  - c. inflate cost figures by multiplying by an appropriate price index.
  - d. adjust cost data by dividing by the percentage rate of inflation.
  
6. An estimated short-run cost function
  - a. would be used to make price and output decisions.
  - b. holds the capital stock constant.
  - c. can be estimated using time-series data.
  - d. all of the above.
  
7. For the short-run cost function  $AVC = a + bQ + cQ^2$ ,
  - a. the  $AVC$  curve is U-shaped when  $a < 0$ ,  $b > 0$ , and  $c < 0$ .
  - b. the  $AVC$  curve is U-shaped when  $a > 0$ ,  $b < 0$ , and  $c > 0$ .
  - c. the corresponding  $SMC$  function is  $SMC = aQ + 2bQ^2 + 3cQ^3$ .
  - d. both  $a$  and  $c$ .
  - e. all of the above.
  
8. A potential problem with cross-section cost data is that
  - a. nominal cost data include the effect of inflation.
  - b. different firms face different input prices.
  - c. at least one input is fixed over time.
  - d. both  $a$  and  $b$ .
  - e. none of the above.

The next six questions refer to the following:

A firm estimated its short-run costs using an average variable cost function of the form

$$AVC = a + bQ + cQ^2$$

and obtained the following results. Total fixed cost is \$1,000.

DEPENDENT VARIABLE:	AVC	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	35	0.8713	108.3	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		43.40	13.80	3.14	0.0036
Q		-2.80	0.90	-3.11	0.0039
Q2		0.20	0.05	4.00	0.0004

9. The estimated marginal cost function is:
  - a.  $SMC = 43.4Q - 1.4Q^2 + 0.07Q^3$
  - b.  $SMC = 43.4 - 1.4Q + 0.07Q^2$
  - c.  $SMC = 43.4Q - 5.6Q^2 + 0.6Q^3$
  - d.  $SMC = 43.4 - 5.6Q + 0.6Q^2$



10. If the firm produces 20 units of output, what is estimated  $AVC$ ?
- \$19.40
  - \$67.40
  - \$171.40
  - \$179.40
11. If the firm produces 20 units of output, what is estimated total cost?
- \$1,348
  - \$1,388
  - \$2,348
  - \$4,428
12. If the firm produces 12 units of output, what is estimated  $SMC$ ?
- \$38.60
  - \$62.60
  - \$105.80
  - \$197.00
13. At what level of output is  $AVC$  minimum?
- 0.14
  - 4.67
  - 7
  - 28
14. What is the minimum value of  $AVC$ ?
- \$ 24.50
  - \$ 33.60
  - \$ 72.80
  - \$121.80
15. A cubic specification for a short-run production function is appropriate when the scatter diagram indicates
- an  $S$ -shaped total product curve.
  - marginal product of labor falls throughout the range of labor usage.
  - total product is decreasing throughout the range of labor usage.
  - an  $S$ -shaped marginal product of labor curve.
  - a  $U$ -shaped marginal product of labor curve.
16. T F With cross-section data it is not necessary to correct for inflation.
17. T F Estimation of a cubic short-run cost function requires that the intercept term be suppressed.
18. T F Input prices are commonly omitted in short-run cost estimation because the span of the time-series data set is generally short enough that real input prices do not change much.
19. T F Once one of the three cost curves  $TVC$ ,  $AVC$ , or  $SMC$  has been estimated, the other two functions cannot be estimated without drawing a new sample of data.

20. T F Short-run cost functions are used by firms to make investment decisions while long-run cost functions provide information for output and pricing decisions.

## Answers

### MATCHING DEFINITIONS

1. empirical production function
2. long-run production function
3. short-run production function
4. cubic production function
5. short-run cubic production function
6. regression through the origin
7. nominal cost data
8. deflating
9. user cost of capital

### STUDY PROBLEMS

1.
  - a. short-run cubic cost function
  - b. short-run cubic marginal cost function
  - c. long-run cubic production function
  - d. short-run cubic average variable cost function
  - e. short-run cubic production function
2.
  - a.  $a > 0, b < 0, c > 0$
  - b. same as part *a*
  - c.  $A = a\bar{K}^3 < 0$  and  $B = b\bar{K}^2 > 0$
  - d. same as part *a*
  - e.  $A < 0, B > 0$
3.
  - a.  $TP = -0.008(15)^3L^3 + 10(15)^2L^2 = -27L^3 + 2,250L^2$   
 $AP = -27L^2 + 2,250L$   
 $MP = 3(-27)L^2 + 2(2,250)L = -81L^2 + 4,500L$
  - b.  $L_m = -B/3A = -2,250/3(-27) = 27.78$  units of labor
  - c.  $L_a = -B/2A = -2,250/2(-27) = 41.67$  units of labor
  - d.  $MP_{L=20} = -81(20)^2 + 4,500(20) = 57,600$   
 $AP_{L=20} = -27(20)^2 + 2,250(20) = 34,200$
4.
  - a.  $Q = -0.05L^3 + 0.6L^2$   
 $AP = -0.05L^2 + 0.6L$   
 $MP = 3(-0.05)L^2 + 2(0.6)L = -0.15L^2 + 1.2L$
  - b. The signs of both parameters are correct: *A* is negative, *B* is positive. The *p*-values indicate significance at better than the 2 percent level for both parameter estimates.
  - c.  $L_a = -B/2A = -0.6/-0.1 = 6$   
 $AP$  reaches its maximum value when 6 units of labor are employed.
  - d.  $Q = -0.05(6)^3 + 0.6(6)^2 = 10.8$   
 At 10.8 units of output,  $AP$  reaches its maximum value.

- e.  $AP_{max} = -0.05(6)^2 + 0.6(6) = 1.8$  (or  $AP_{max} = Q/L = 10.8/6 = 1.8$ )  
 So,  $AVC = w/AP = 36/1.8 = \$20$   
 Since  $AP = MP$  when  $AP$  is at its maximum value,  $AVC = SMC = \$20$  at  $L = 6$  and  $Q = 10.8$ .
- f. When  $L = 7$ ,  $AP = 1.75$  and  $MP = 1.05$ . Thus,  $AVC = 36/1.75 = \$20.57$  and  $SMC = 36/1.05 = \$34.29$ .
5. a. The parameter restrictions are:  $a > 0$ ,  $b < 0$ , and  $c > 0$ . In each case, the absolute value of the  $t$ -ratio is greater than the critical value of 2.160.
- b.  $SMC = 75 - 4.8Q + 0.18Q^2$
- c.  $TVC = 75Q - 2.4Q^2 + 0.06Q^3$
- d.  $Q_m = -b/2c = 2.4/0.12 = 20$
- f.  $AVC_{min} = 75 - 2.4(20) + 0.06(20)^2 = 51$

### **COMPUTER PROBLEM**

1. Yes,  $\hat{A} < 0$  and  $\hat{B} > 0$ . Both  $\hat{A}$  and  $\hat{B}$  are statistically significant at better than the 1 percent level. The estimated model explained only about 62 percent of the variation in output. The computer printout looks like this:

DEP. VARIABLE:	Q	R-SQUARE	F-RATIO	P-VALUE ON F	
OBS:	22	0.6198	83.72	0.0000	
VARIABLE	PARAMETER ESTIMATE	STD. ERROR	T-RATIO	P-VALUE	
L3	-0.04249	0.01491	-2.85	0.0099	
L2	2.77199	0.55584	4.99	0.0001	

2.  $\hat{Q} = -0.04249L^3 + 2.77199L^2$   
 $AP = -0.04249L^2 + 2.77199L = \hat{A}L^2 + \hat{B}L$   
 $MP = -0.12747L^2 + 5.54398L = 2\hat{A}L^2 + 3\hat{B}L$
3.  $L_a = -B/2A = -(2.77199)/(2 \times -0.04249) = 32.62$  workers per day  
 $AP(L_a) = -0.04249(32.62)^2 + 2.77199(32.62) = 45.21$  carts per worker  
 $AVC = w/AP = \$160/45.21 = \$3.54$  per cart
4. At maximum  $AP$ ,  $AP = MP$ , so  $SMC = AVC$ , and thus  $SMC = \$3.54$ . You can verify this result by noticing that  $MP$  (at  $L = 32.62$ ) is 45.21. Thus,  $SMC = w/MP = \$160/45.21 = \$3.54$ , which is exactly the value found in question 3 for  $AVC$ .
5.  $L_m = 21.75$  workers per day;  $\hat{Q}(21.75) = 874$  carts per day

### **MULTIPLE CHOICE / TRUE-FALSE**

1. e Empirical production and cost functions are all of these things.
2. d To estimate a short-run production function, at least one input must be fixed; that is, usage of one of the inputs must take the same value for each observation in the sample. A time-series on the same firm is usually the best way to accomplish this.
3. c An S-shaped total variable cost function requires a cubic specification.

4. e The user cost of capital accounts for the cost of acquiring capital, and also any depreciation or capital gains/losses resulting from using and owning capital. The user cost of capital also includes the opportunity cost of using its capital rather than renting it.
5. b Nominal dollars are adjusted for the effects of inflation by dividing the nominal dollars by a price index to get real (or constant) dollars.
6. d All of these statements are true in general.
7. b See Table 10.3 in your textbook.
8. b If input prices vary, then they must be included in the model as explanatory variables.
9. d  $SMC = a + 2bQ + 3cQ^2$ , where  $a = \text{intercept} = 43.40$ ,  $2b = 2 \times -2.80 = -5.6$ , and  $3c = 3 \times 0.20 = 0.60$ .
10. b  $AVC_{Q=20} = 43.40 - 2.80 \times 20 + 0.20 \times 20^2 = \$67.40$
11. c  $TC_{Q=20} = (AVC_{Q=20} \times 20) + TFC = 67.40 \times 20 + 1,000 = \$2,348$
12. b  $\$62.60 = SMC_{Q=12} = 43.40 - 5.60 \times 12 + 0.60 \times 0.12^2$
13. c  $AVC_{min} = -b/2c = -(-2.8)/(2 \times 0.2) = 7$
14. b  $AVC_{Q=7} = \$33.60 = 43.40 - 2.80 \times 7 + 0.20 \times 7^2 = \$67.40$
15. a A cubic specification has an S-shape.
16. T Correcting for inflation is not necessary because all data are for the same period in time.
17. F Regression through the origin is not employed in estimating a cubic short-run *cost* equation but rather in estimating a cubic short-run *production* function.
18. T If inflation affects all input prices equiproportionately, real input prices will not vary.
19. T Knowing any one equation allows the other two to be derived mathematically.
20. F Long-run cost data are used for investment decisions, while short-run cost data are used for output and pricing decisions.

## Homework Exercises

1. A firm estimates its cubic production function of the following form:

$$Q = AL^3 + BL^2$$

and obtains the following results:

DEPENDENT VARIABLE:	Q	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	32	0.7547	92.31	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
L3		-0.0016	0.0005	-3.20	0.0032
L2		0.4000	0.0950	4.21	0.0002

- a. The equations for total product, average product, and marginal product are:  
 $TP =$  \_\_\_\_\_  
 $AP =$  \_\_\_\_\_  
 $MP =$  \_\_\_\_\_
- b. The estimated values of  $A$  and  $B$  are statistically significant at the (exact) levels, \_\_\_\_\_ and \_\_\_\_\_, respectively.
- c. At \_\_\_\_\_ units of labor usage, marginal product of labor begins to diminish.

When the wage rate is \$300, answer the following questions. (Remember that  $AP = Q/L$ ;  $AVC = w/AP$ ; and  $SMC = w/MP$ .)

- d. Average product of labor reaches its maximum value at \_\_\_\_\_ units of labor.
- e. At the output for part  $d$ , average variable cost is \$\_\_\_\_\_ and marginal cost is \$\_\_\_\_\_.
- f. When the rate of labor usage is 100 units of labor, output is \_\_\_\_\_ units. Average variable cost is \$\_\_\_\_\_ and marginal cost is \$\_\_\_\_\_.

2. Suppose Heritage Corporation believes that its total variable costs follow a cubic specification and so it estimates its average variable costs using the following specification:

$$AVC = a + bQ + cQ^2$$

The regression analysis produces the following computer output:

DEPENDENT VARIABLE:	AVC	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	45	0.6145	33.47	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		175.0	25.00	7.00	0.0001
Q		-3.20	0.80	-4.00	0.0003
Q2		0.08	0.01	8.00	0.0001

- a. Do the estimated coefficients have the required signs to yield a U-shaped  $AVC$  curve? Discuss the significance using the  $p$ -values.

- b. Heritage Corporation's marginal cost function is

$$SMC = \underline{\hspace{10em}}$$

- c. At what level of output does  $AVC$  reach a minimum? What is the value of  $AVC$  at its minimum?

$$Q_{min} = \underline{\hspace{2em}} \quad AVC_{min} = \underline{\hspace{10em}}$$

- d. Compute  $AVC$  and  $SMC$  when Heritage produces 8 units.

$$AVC_{Q=8} = \underline{\hspace{2em}}$$

$$SMC_{Q=8} = \underline{\hspace{2em}}$$

3. **COMPUTER EXERCISE**

Use a computer regression package or Excel to work this computer exercise.

Palm Products Company has collected data on its average variable costs of production for the past 12 months. The costs have been adjusted for inflation by deflating with an appropriate price index. The *AVC* and associated output data are presented below:

<i>obs</i>	<i>Q</i>	<i>AVC</i>	<i>obs</i>	<i>Q</i>	<i>AVC</i>
1	22	\$208	7	45	\$172
2	31	202	8	45	158
3	31	206	9	45	173
4	25	214	10	62	170
5	41	174	11	62	152
6	41	203	12	70	175

- Run the appropriate regression to estimate the parameters for the empirical cost function  $AVC = a + bQ + cQ^2$ .
- Using a 10 percent significance level, discuss suitability of the parameter estimates obtained in part *a*. Consider both the algebraic signs and statistical significance of the parameter estimates.

- Present the estimated average variable cost, total variable cost, and short-run marginal cost functions.

- At what level of output does *AVC* reach its minimum value? What is the minimum value of *AVC* at its minimum?

$$Q_{\min} = \underline{\hspace{2cm}} \quad AVC_{\min} = \underline{\hspace{2cm}}$$

- Compute *AVC* and *SMC* when Palm Products produces 20 units of output:

$$AVC_{Q=20} = \underline{\hspace{2cm}}$$

$$SMC_{Q=20} = \underline{\hspace{2cm}}$$

Is *AVC* rising or falling when Palm produces 20 units? Explain.

- At what level of output does *SMC* equal *AVC*? How did you get this answer?